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Tree-Constrained Matching

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[3] The University of Sydney, Australia

ICALP'11

Analysis of Live Cell Video

Given live cell video, we want to track individual cells

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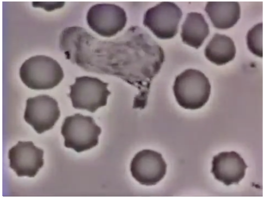

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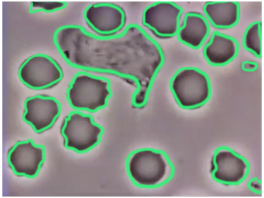

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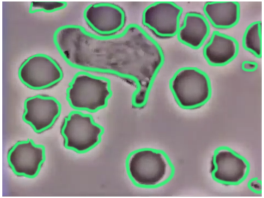

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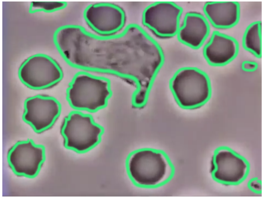

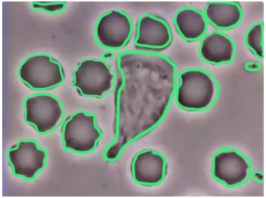

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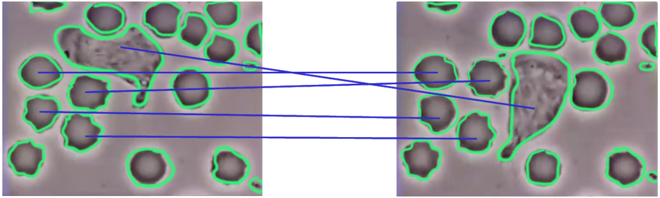

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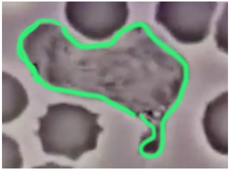

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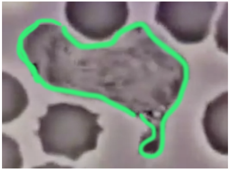

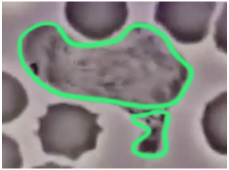

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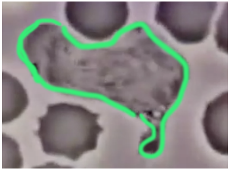

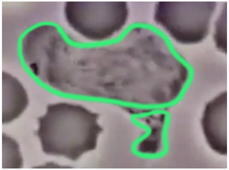

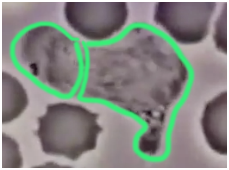

Over/under segmentation

Challenge: Biological cell division vs. over-segmentation

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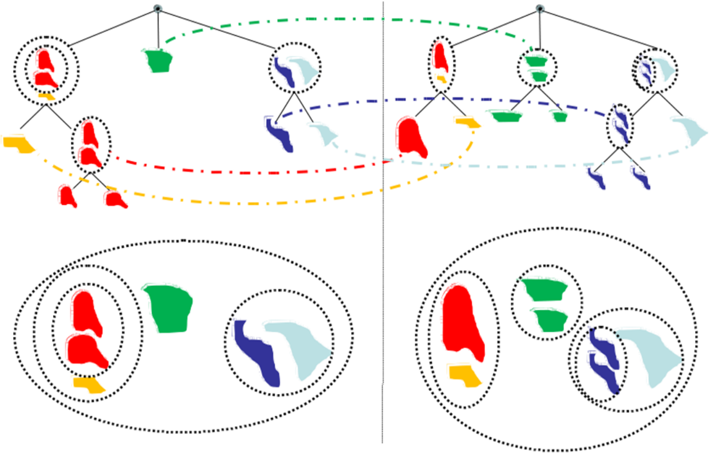
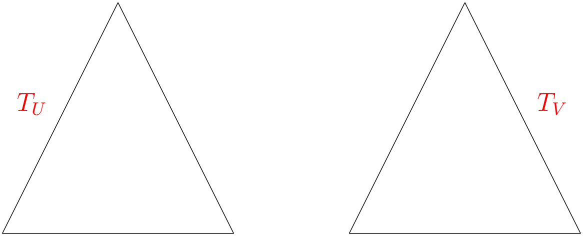
⇒ [Mosig *et al.*, 2009]: Integrate identification and tracking steps!

Cosegmentation



Tree-Constrained Matching



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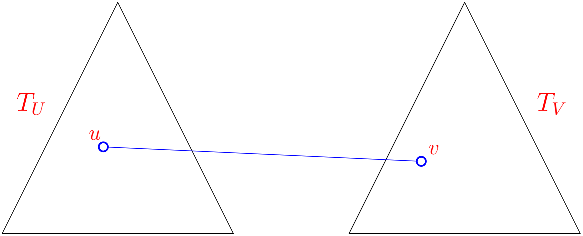

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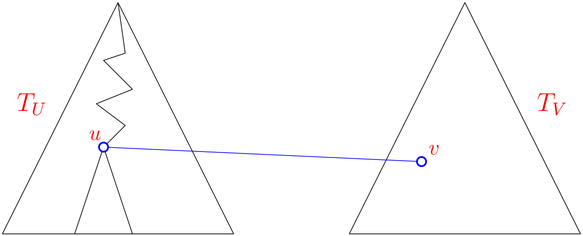

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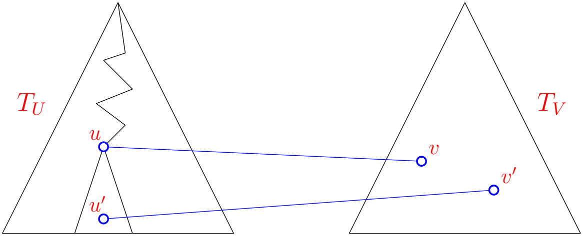

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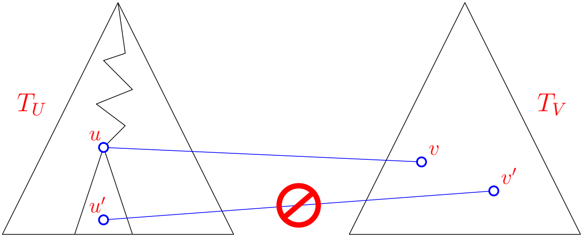

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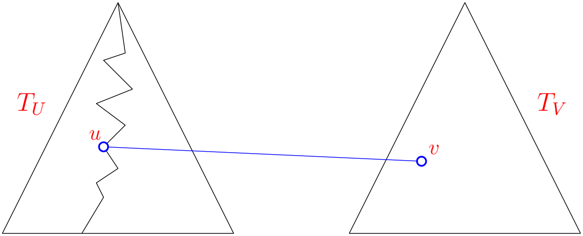

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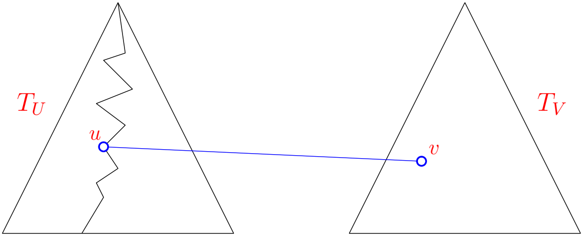

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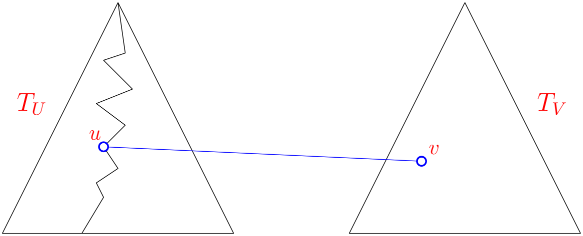

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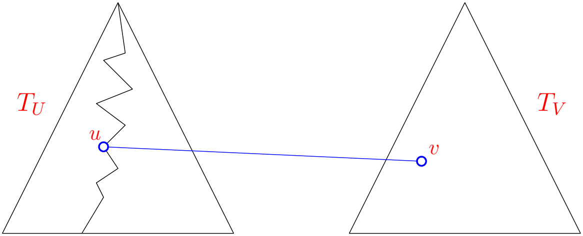

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Unfortunately, as we shall see, this is not the case

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Thus, there is a 4-approximation for TCM

Our Results

Show that TCM is APX-hard and disprove claim of Mosig *et al.*

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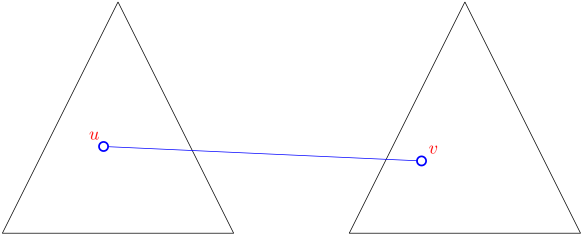
Generalization to posets

Reducing TCM to MIS in 2-IG

Every edge (u, v) is assigned one interval in T_U and one in T_V



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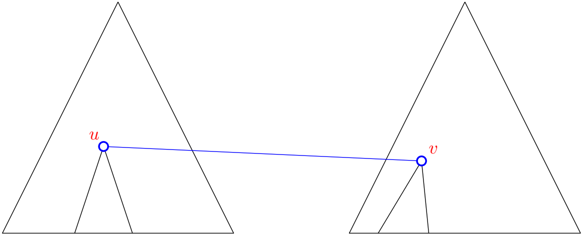

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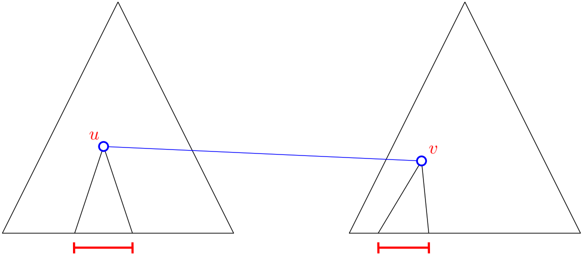

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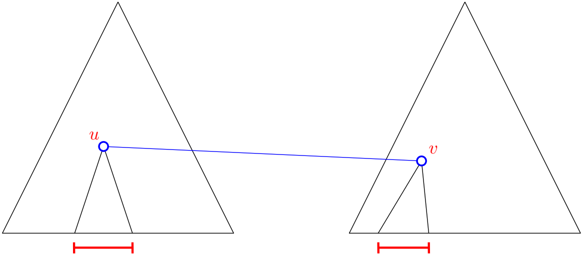
Every edge (u, v) is assigned one interval in T_U and one in T_V

Matching is feasible $\Leftrightarrow$ corresp. set of 2-intervals is independent



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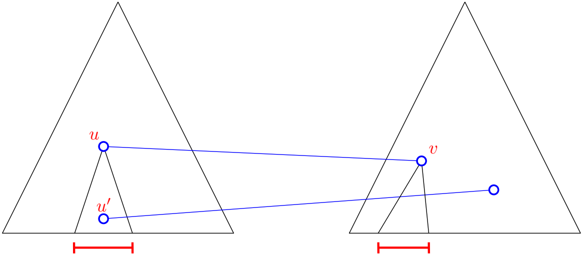

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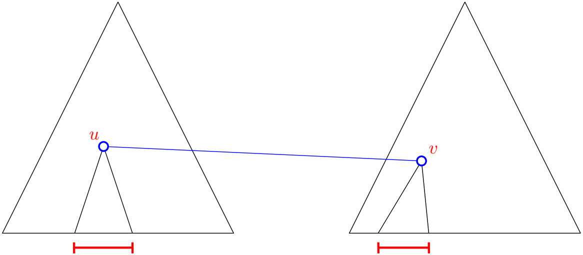

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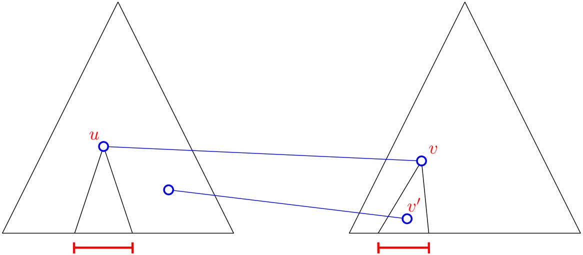

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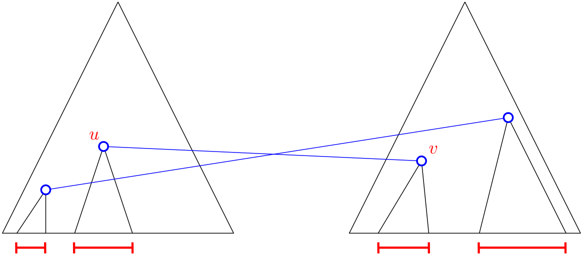

Bar-Yehuda *et al.* algorithm

$$\max \sum_{u \in V} x_u$$

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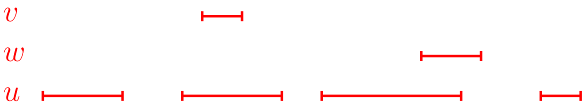
MIS-d-interval(G)

1: let x be optimal LP solution
2: let $\mathcal{S}$ be the empty set
3: **while** G is not empty **do**
4: let u minimize $x(N(u))$
5: add u to $\mathcal{S}$
6: remove $N(u) + u$ from G
7: **return** $\mathcal{S}$

∀ feasible x: $\exists u : x(N(u)) \leq 2d$

$$\sum_{u} x_u \sum_{v \in N(u)} x_v$$



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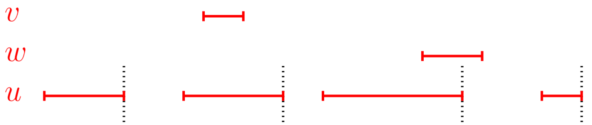

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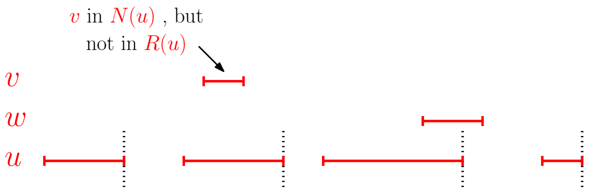

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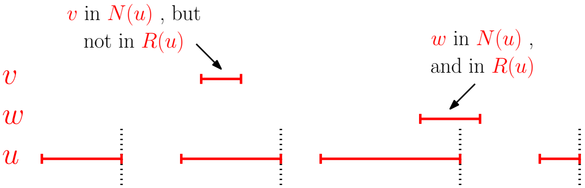

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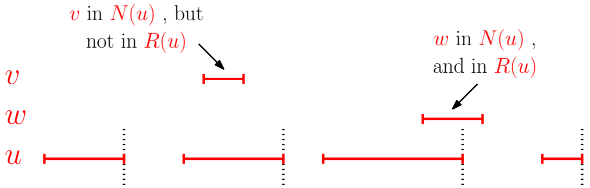
$$\sum_{u} x_u \sum_{v \in N(u)} x_v \leq 2 \sum_{u} x_u \sum_{v \in R(u)} x_v$$



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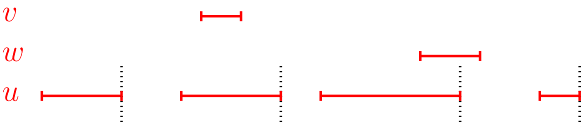

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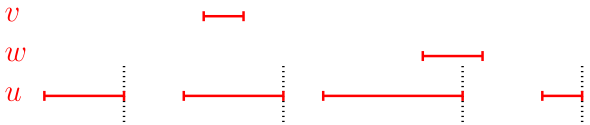

4-approximation for TCM

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$$\text{s.t.} \sum_{e \text{ on } P} x_e \leq 1 \quad \forall \text{desc. path } P$$

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TCM(U,V,E)

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let x be optimal LP solution
let 𝓜 be the empty set
while E is not empty do
    let e minimize x(N(e))
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return 𝓜
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$N(e)$ is the set of edges in conflict with e

TCM(U,V,E)

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$\forall$ basic feasible x: $\exists e : x(N(e)) \leq 3$

For ease of analysis, assume that

- For all edges $x_e > 0$
- No leaf is unmatched

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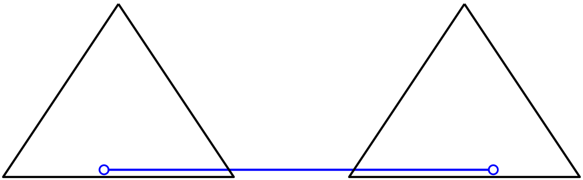

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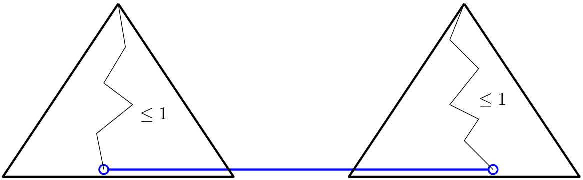

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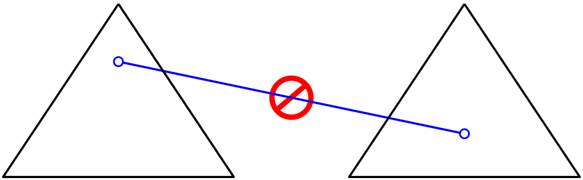

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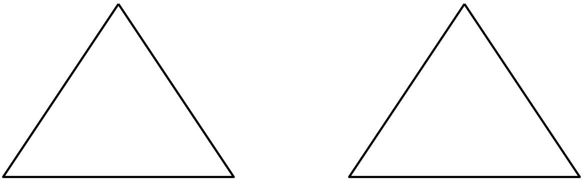

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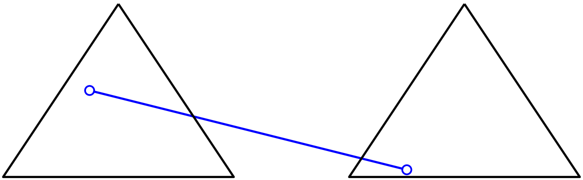

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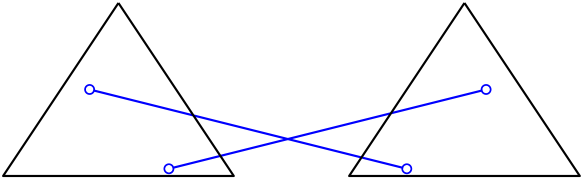

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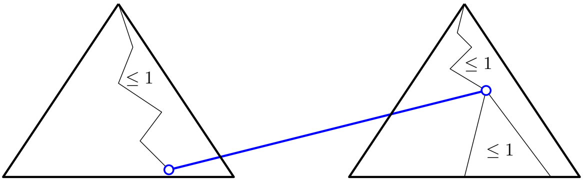
For ease of analysis, assume that

- For all edges $x_e > 0$
- No leaf is unmatched

If we have a leaf-to-leaf edge, we are done

Otherwise, no internal-to-internal edges



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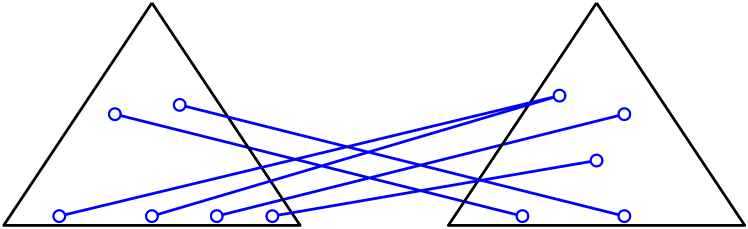
Integrality gap of LP formulation is $2 - o(1)$

2-approximation

Idea: Exploit bfs structure if $\forall e : x(N(e)) > 2$



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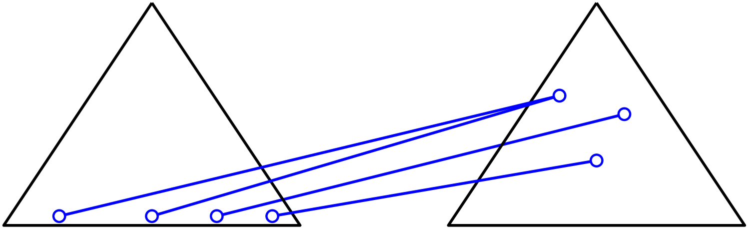

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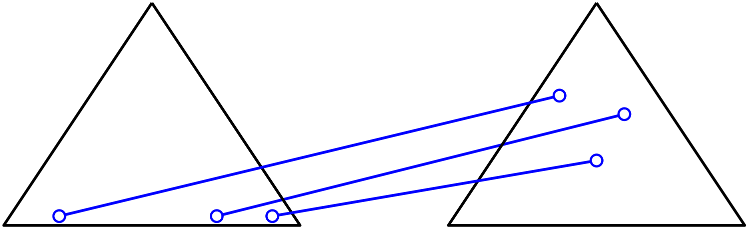

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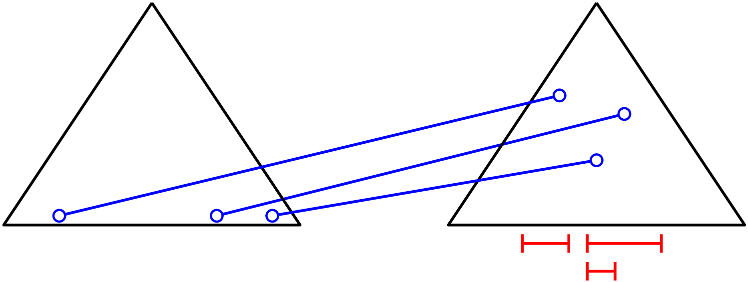

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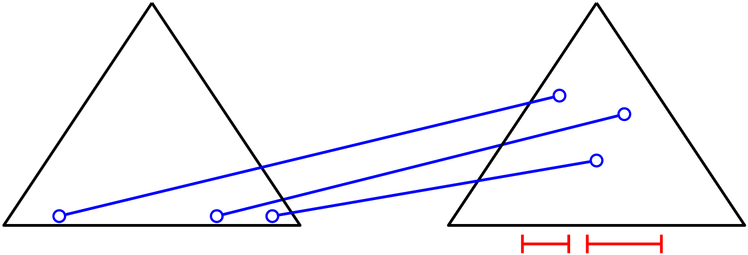

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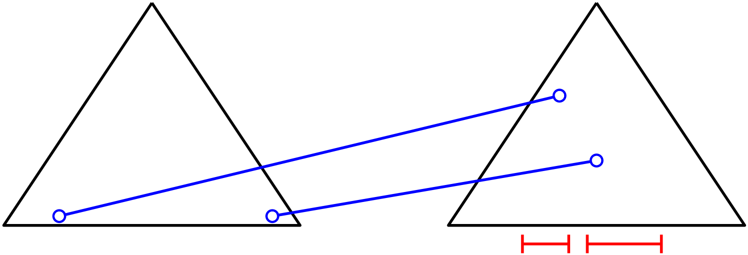

Generalization to posets

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It cannot be approximated to $2^{\log^{1-\epsilon} \rho}$, for any $\epsilon > 0$, unless $NP \subseteq DTIME(n^{\text{polylog}\, n})$

Thank you for your attention!