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Tree-Constrained Matching

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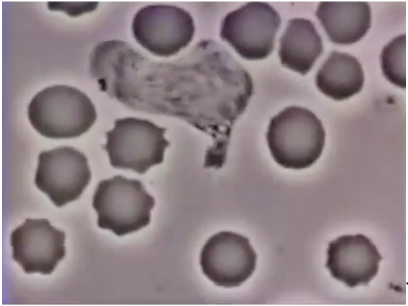
[3] The University of Sydney, Australia

November 29, 2011

Analysis of Live Cell Video



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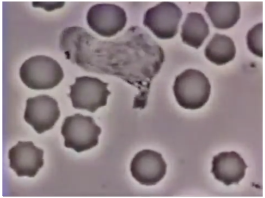

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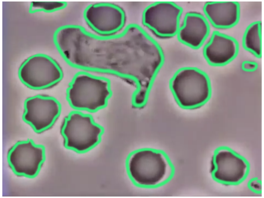

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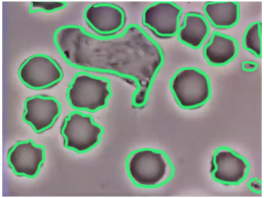

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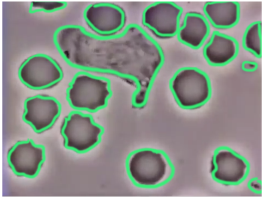

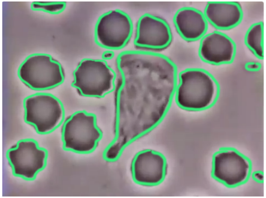

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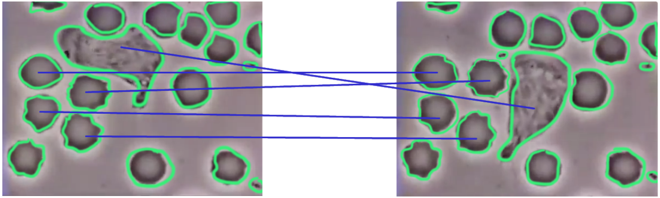
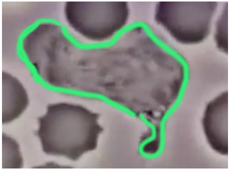


Over/under segmentation



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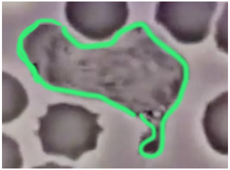 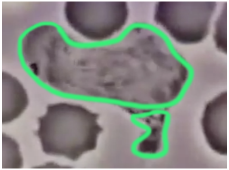

Over/under segmentation

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Challenge: Biological cell division vs. over-segmentation

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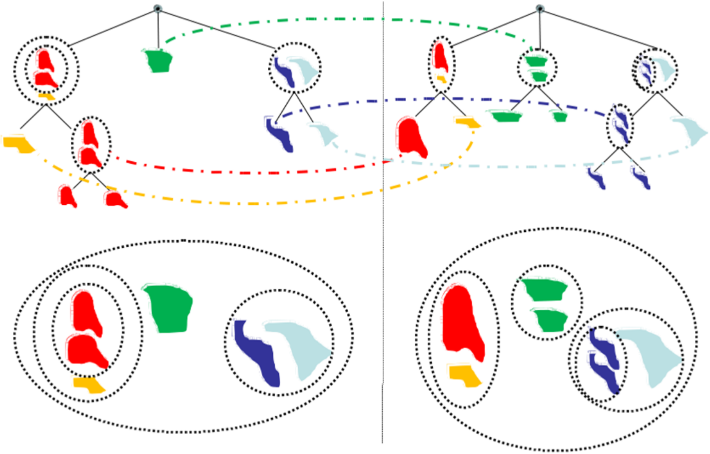
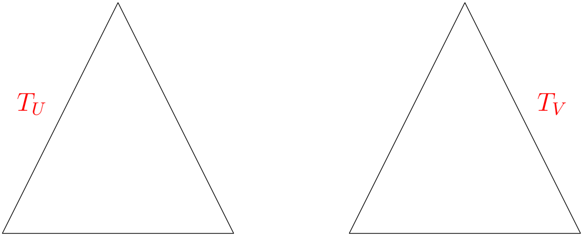
⇒ [Mosig *et al.*, 2009]: Integrate identification and tracking steps!

Cosegmentation



Tree-Constrained Matching



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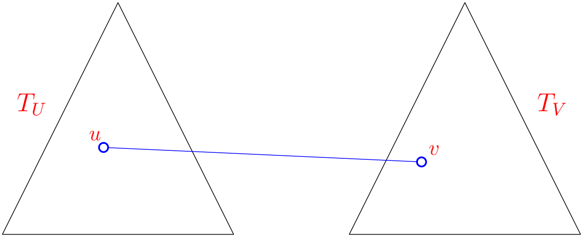

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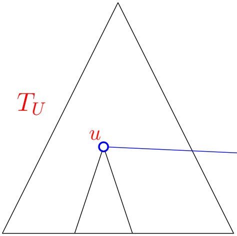

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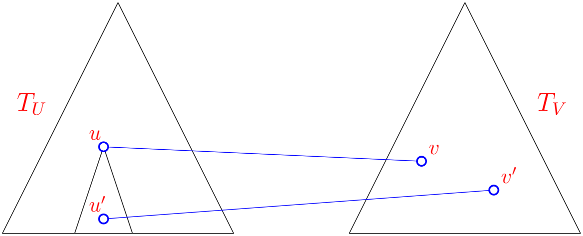

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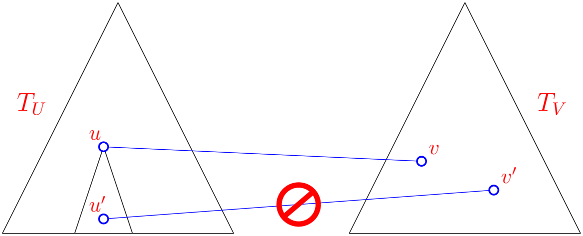

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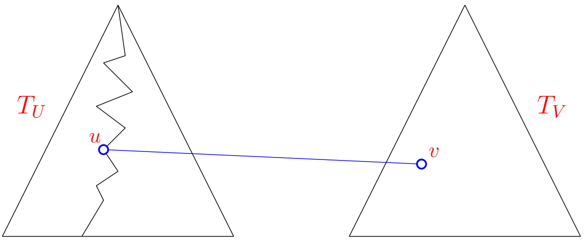

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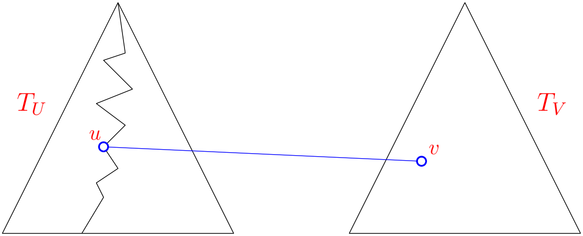

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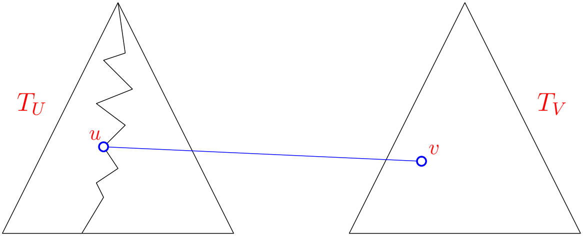

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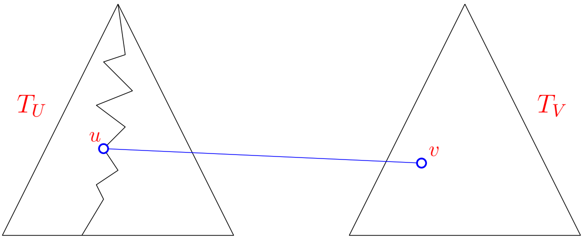

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Unfortunately, as we shall see, this is not the case

MIS in d-Interval Graphs

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- If $d = 1$, there is an exact algorithm
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Thus, there is a 4-approximation for TCM

Our Results

Show that TCM is APX-hard and disprove claim of Mosig *et al.*

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Give 2-approximation, matching integrality gap of LP formulation

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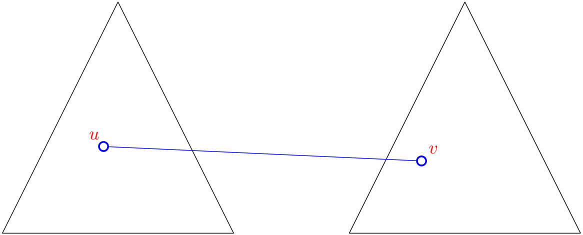
Generalization to posets

Reducing TCM to MIS in 2-IG

Every edge (u, v) is assigned one interval in T_U and one in T_V



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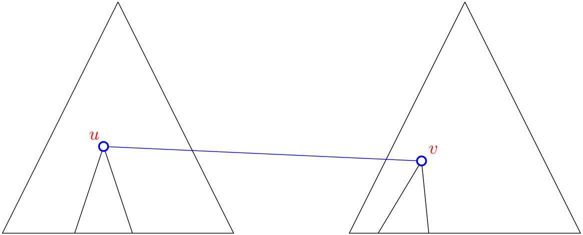

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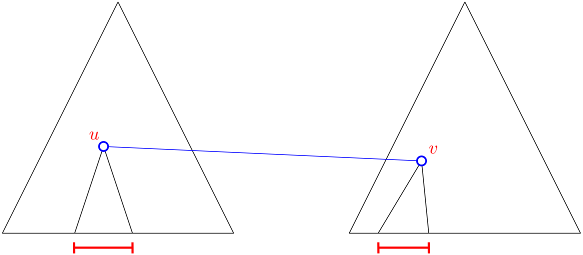

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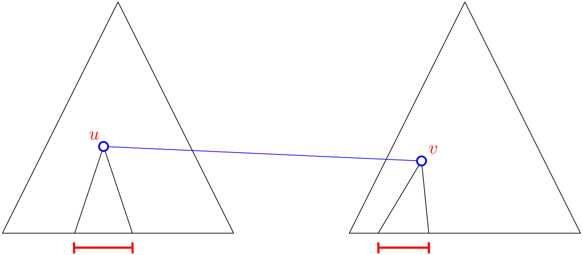
Every edge (u, v) is assigned one interval in T_U and one in T_V

Matching is feasible $\Leftrightarrow$ corresp. set of 2-intervals is independent



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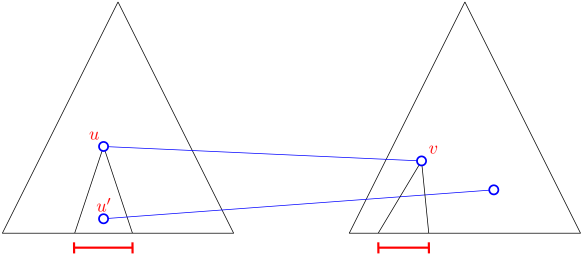

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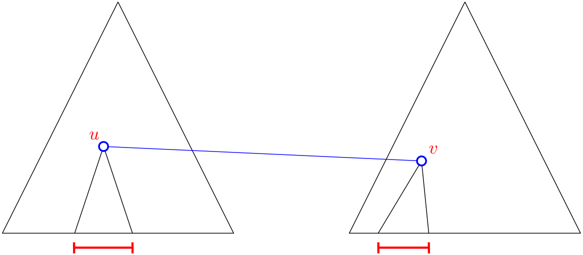

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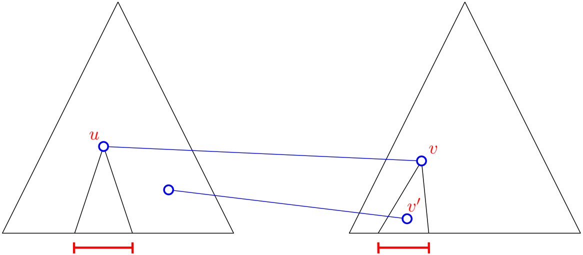

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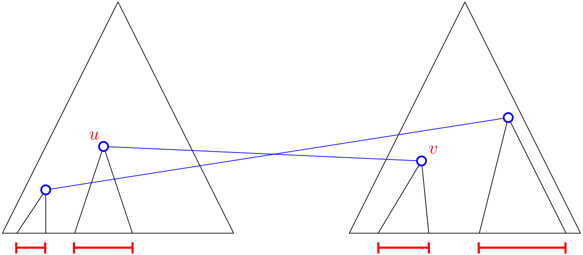
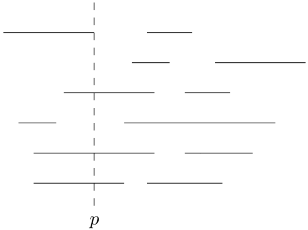


Bar-Yehuda *et al.* algorithm



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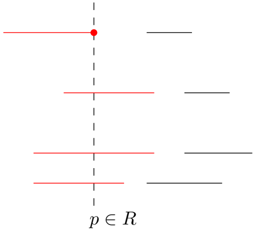

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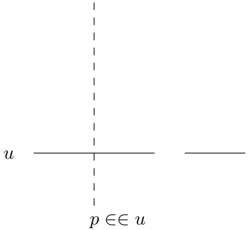

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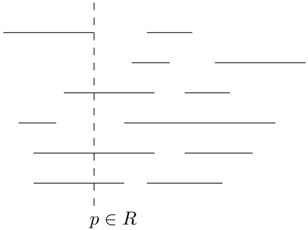
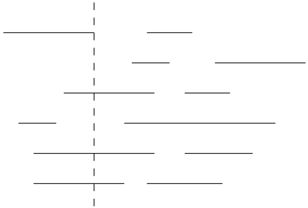


$$\max \sum_{u \in V} x_u$$

$$\text{s.t.} \sum_{u:p \in \in u} x_u \leq 1 \quad \forall p \in R$$

$$x_u \geq 0 \quad \forall d\text{-interval } u$$

Bar-Yehuda *et al.* algorithm



$$x(V') := \sum_{u \in V'} x_u$$

$$\begin{aligned} \max \quad & \sum_{u \in V} x_u \\ \text{s.t.} \quad & \sum_{u : p \in \in u} x_u \leq 1 \quad && \forall p \in R \\ & x_u \geq 0 && \forall d\text{-interval } u \end{aligned}$$

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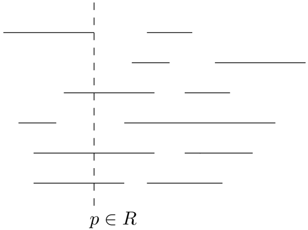

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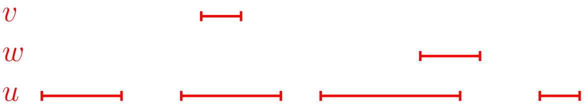
MIS-d-interval(G)

```
let x be optimal LP solution
let S be the empty set
while G is not empty do
    let u minimize x(N[u])
    add u to S
    remove N(u)+u from G
return S
```

$\forall$ feasible x: $\exists u : x(N[u]) \leq 2d$

$$\sum_u x_u \sum_{v \in N[u]} x_v$$



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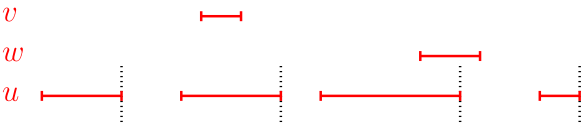

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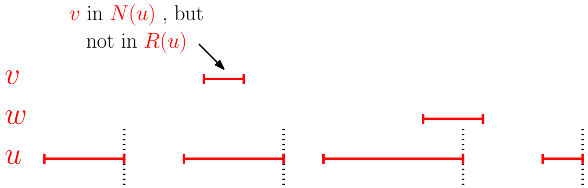

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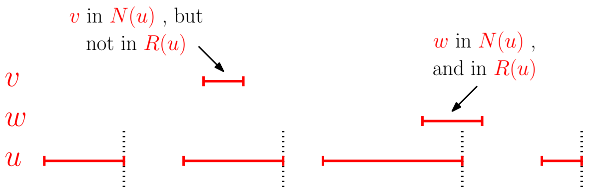
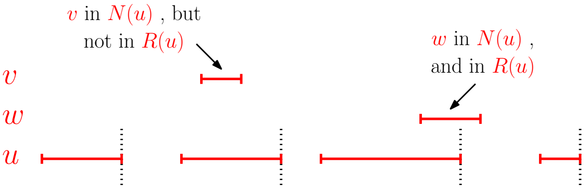


∀ feasible x: $\exists u : x(N[u]) \leq 2d$

$$\sum_{u} x_u \sum_{v \in N[u]} x_v \leq 2 \sum_{u} x_u \sum_{v \in R(u)} x_v$$



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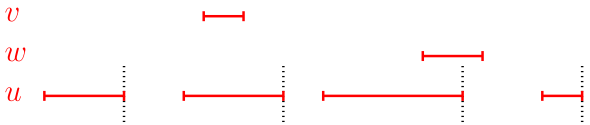
$$\leq 2 \sum_u x_u \, d$$



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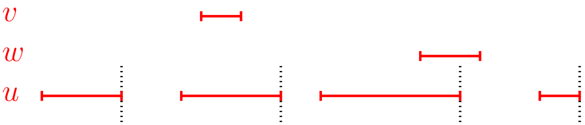

4-approximation for TCM

$$\begin{aligned} \max \sum_{e \in E} x_e & \\ \text{s.t.} \sum_{e \text{ on } P} x_e \leq 1 & \quad \forall \text{desc. path } P \\ x_e \geq 0 & \quad \forall \text{ edge } e \end{aligned}$$

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TCM(U,V,E)

let x be optimal LP solution
let $\mathcal{M}$ be the empty set
while E is not empty **do**
    let e minimize $x(N[e])$
    add e to $\mathcal{M}$
    remove $N[e] + e$ from E
return $\mathcal{M}$

4-approximation for TCM

$$\max \sum_{e \in E} x_e$$

$$\text{s.t.} \sum_{e \text{ on } P} x_e \leq 1 \qquad \forall \text{desc. path } P$$

$$x_e \geq 0 \qquad \forall \text{ edge } e$$

$N[e]$ is the set of edges in conflict with e

TCM(U,V,E)

```
let x be optimal LP solution
let 𝓜 be the empty set
while  E is not empty do
    let e minimize x(N[e])
    add e to 𝓜
    remove N[e] + e from E
return 𝓜
```

$\forall$ basic feasible x: $\exists e : x(N[e]) \leq 3$

For ease of analysis, assume that

- For all edges $x_e > 0$
- No leaf is unmatched

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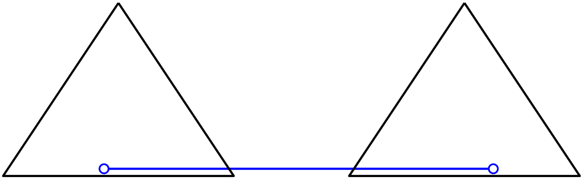

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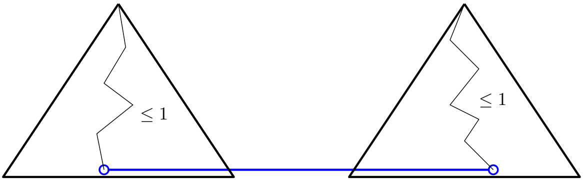

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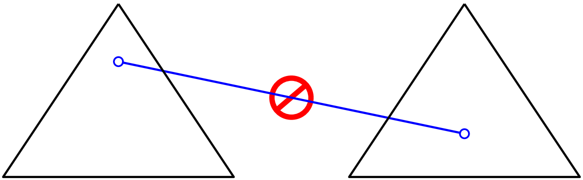
Otherwise, no internal-to-internal edges



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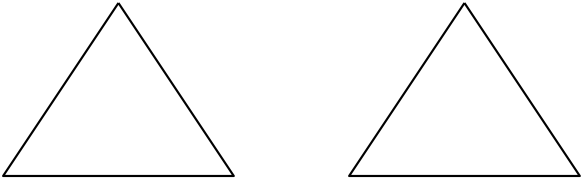

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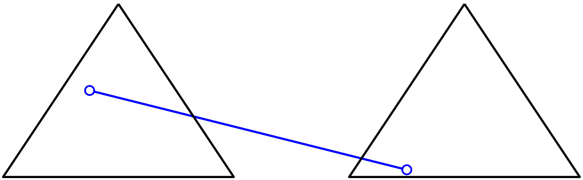

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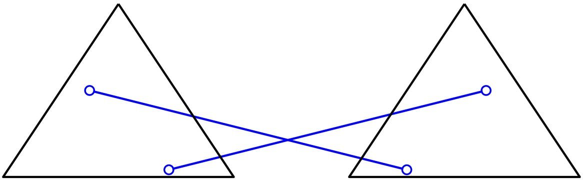

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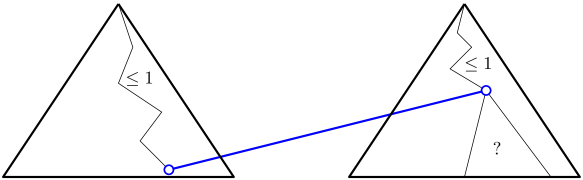

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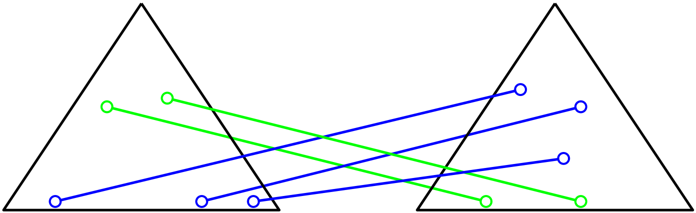

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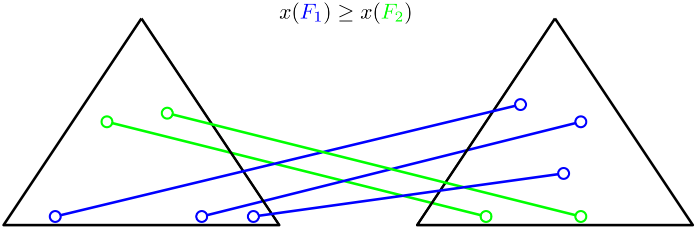

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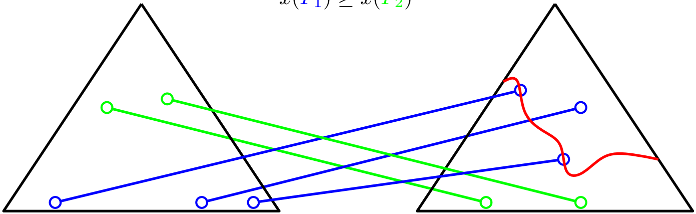

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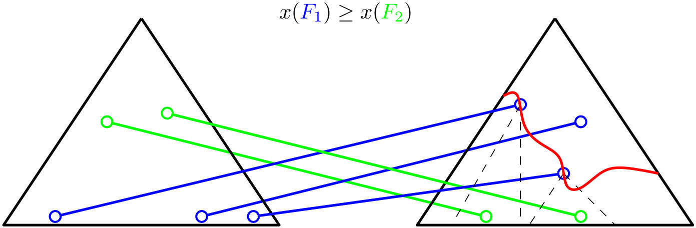

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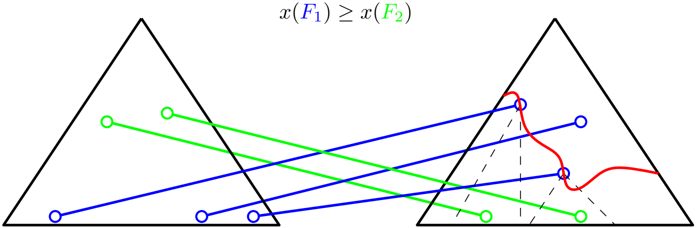

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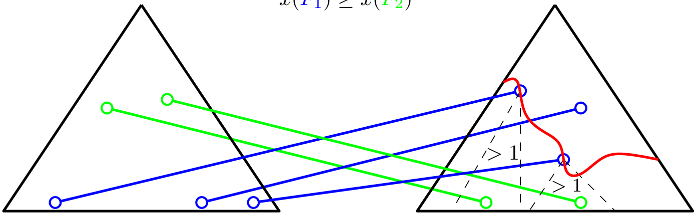

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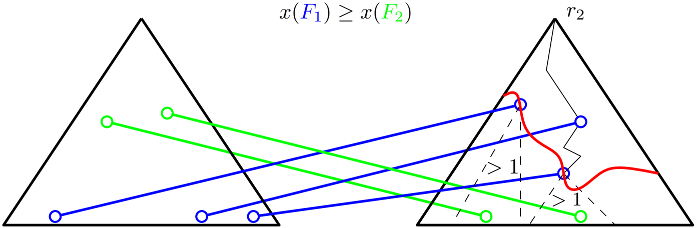

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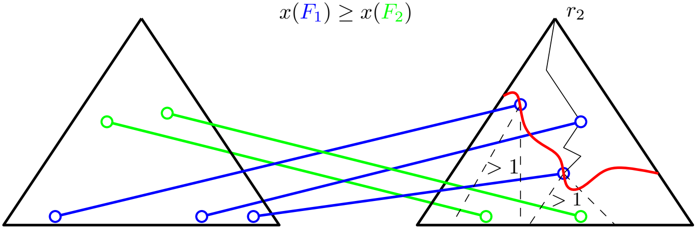

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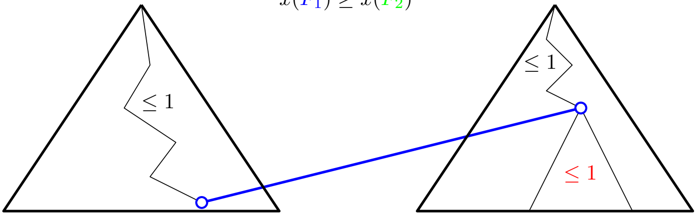

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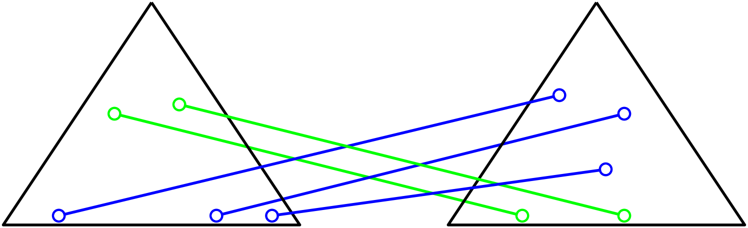
Integrality gap of LP formulation is $2 - o(1)$

2-approximation

Idea: Exploit bfs structure if $\forall e : x(N[e]) > 2$



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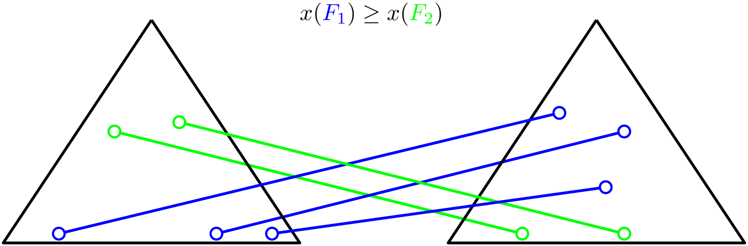

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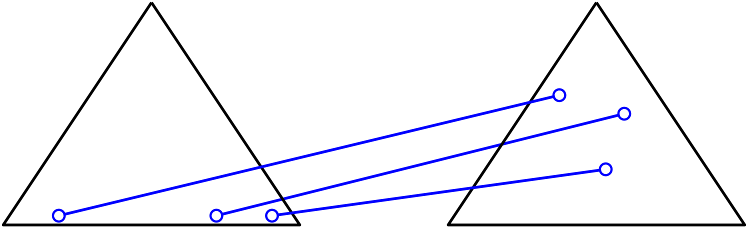

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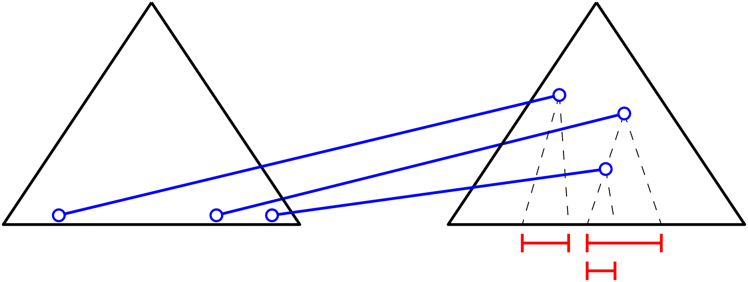

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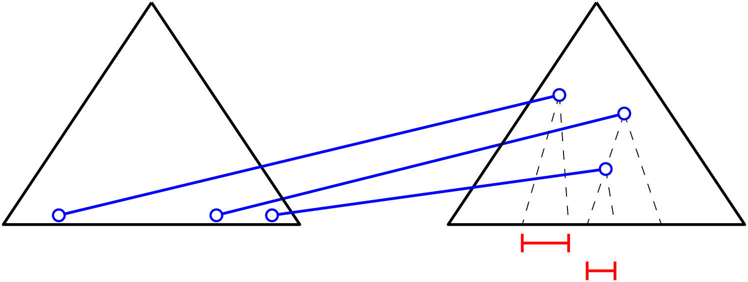

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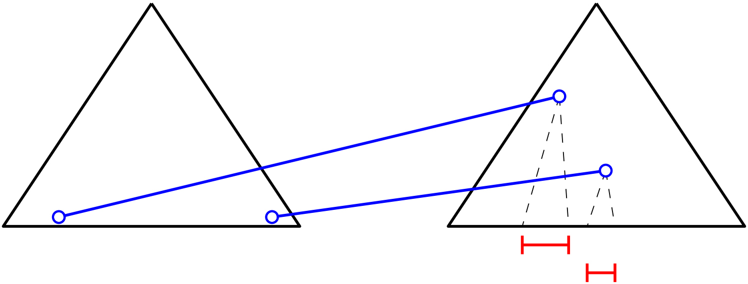

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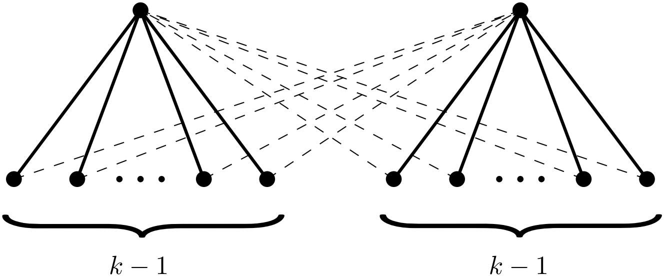

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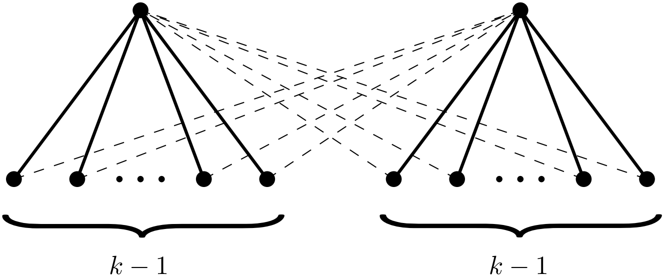

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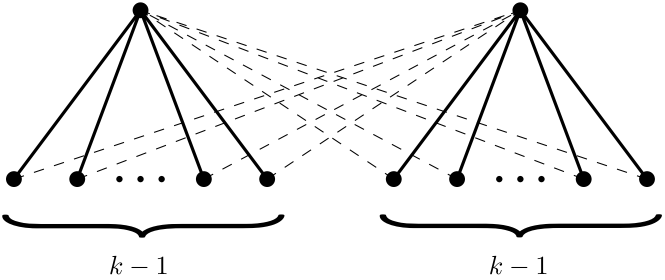

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Independence constraints on perfect graphs

Thank you for your attention!